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# GROUPS GENERATING TRANSVERSALS TO SEMISIMPLE LIE GROUP ACTIONS

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ABSTRACT

We describe those discrete groups with finite measure preserving actions that are stably orbit equivalent to such an action of a higher rank simple Lie group. This is applied to obtain information on the question of when ergodic equivalence relations are generated by a free action of a group.

## 1. Statement of main results

The orbit structure of finite measure preserving actions of simple non-compact Lie groups exhibits a wide range of very strong rigidity properties ([4], [6], [7], for example), which are closely related to various questions concerning transversal structures on certain foliations [8], [9]. The main result of this paper is to establish another such result.

THEOREM 1.1. *Let $G$ be a connected, simple Lie group with finite center and* $\mathbf{R}$-rank$(G) \geq 2$, *and suppose $M$ is an essentially free ergodic $G$-space with a finite invariant measure. Let $\Gamma$ be a countable group, and suppose $X$ is an essentially free, ergodic $\Gamma$-space with finite invariant measure. Assume the $G$-action on $M$ and the $\Gamma$-action on $X$ are stably orbit equivalent. If the group $\Gamma$ admits a non-finite linear representation $\Gamma \to \mathrm{GL}(n,\mathbf{C})$ for some $n$, then there is a subgroup $\Gamma' \subset \Gamma$ of finite index and a finite normal subgroup $N \subset \Gamma'$ for which $\Gamma'/N$ is isomorphic to a lattice in* $\mathrm{Ad}(G)$.

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(By a finite representation of a group we mean one that factors through a finite quotient of the group. For various measure theoretic notions concerning orbit equivalence, stable orbit equivalence, etc., we refer to [2] and [3].)

COROLLARY 1.2. *Let $G$ be as in Theorem* 1.1 *and $\Lambda \subset G$ a lattice. Suppose $\Gamma$ is a countable group with a non-finite (finite dimensional) linear representation. If $\Lambda$ and $\Gamma$ have essentially free finite measure-preserving ergodic actions that are orbit equivalent, then there is a subgroup $\Gamma' \subset \Gamma$ of finite index and a finite normal subgroup $N \subset \Gamma'$ for which $\Gamma'/N$ is isomorphic to a lattice in* $\mathrm{Ad}(G)$.

COROLLARY 1.3. *With the assumptions of Theorem* 1.1 *or Corollary* 1.2, *make the further assumption that $\Gamma$ is residually finite, e.g., $\Gamma$ admits a faithful finite-dimensional linear representation. Then there is a subgroup of $\Gamma$ of finite index that is isomorphic to a lattice in* $\mathrm{Ad}(G)$.

Theorem 1.1 is also related to a classical problem formulated by Feldman-Moore in [3]. Namely, they prove in [3] that any countable Borel equivalence relation on a standard Borel space is given by the orbits of some countable group action. They then ask whether or not one can always find a free action of some countable group whose orbits are precisely the equivalence classes. A counterexample was given by S. Adams in [1]. However, this example left open the measure theoretic version of the question, namely whether or not every countable Borel equivalence relation with an ergodic finite invariant measure could be given (a.e.) by an essentially free action of a countable group. Theorem 1.1, combined with the results on orbit equivalence in [6] and Witte's results in [5], yields ergodic equivalence relations which cannot be given by the orbits of an essentially free action of a countable group which admits a non-finite linear representation. Namely:

COROLLARY 1.4. (a) *Let $G$ be as in Theorem* 1.1 *with trivial center and suppose $H$ is another connected simple Lie group with trivial center for which we have $G \subset H$ as a proper subgroup. Let $\Lambda \subset H$ be a lattice, and let $G$ act on $H/\Lambda$. Let $T \subset H/\Lambda$ be a transversal to this action, and assume that $T$ has finite measure with respect to the natural measures on transversals.* (*See* [2].) *Then the countable ergodic equivalence relation on $T$ is not given by an essentially free action of any countable group which admits a finite-dimensional non-finite linear representation over* $\mathbf{C}$.

(b) *Let $G$ be as in Theorem* 1.1 *and let $\Lambda \subset G \times G$ be an irreducible lattice. We can then view $\Lambda$ as a dense subgroup of $G$ by projection onto either of the coordinates. Let $A \subset G$ be a set of finite positive measure, and consider the equivalence relation on $A$ given by intersection with the left cosets of $\Lambda$. Then this countable* *ergodic equivalence relation on A is not given by an essentially free action of any countable group which admits a finite-dimensional non-finite linear representation over* **C**.

This result of course highlights the question as to whether one can remove the condition that the group admits a finite-dimensional non-finite linear representation over **C**. If so, one has a negative answer to the measure theoretic Feldman-Moore question. On the other hand, if not, and such groups exist, one then has groups naturally associated to a simple Lie group $G$ which are not linear, but which have a number of properties of lattices. (For example, one can show that for such groups, normal subgroups are either finite or of finite index.)

Here is an application to foliations with a transverse geometric structure, which can also be deduced from the results of [8].

COROLLARY 1.5. *Let $G$ be as in Theorem* 1.1, *and let $Y$ be the associated symmetric space. Let $M$ be a manifold with either a locally free, essentially free, ergodic $G$-action or an ergodic foliation with leaves locally isometric to $Y$ and almost every leaf simply connected. Let $\Gamma$ be an infinite countable group acting isometrically and essentially freely, on a simply connected compact Riemannian manifold $T$. Assume $\Gamma$ admits a faithful linear representation. If the action or foliation admits a transverse $(\Gamma, T)$-structure, then the holonomy group (i.e., the image of the holonomy homomorphism $\pi_1(M) \to \Gamma$) has a subgroup of finite index isomorphic to a lattice in* $\mathrm{Ad}(G)$.

## 2. Proof of Theorem 1.1

We let $G$, $M$, $\Gamma$, and $X$ be as in the statement of the theorem. Since the actions are stably orbit equivalent, we may choose (possibly after discarding a $\Gamma$-invariant Borel null set) an injective Borel map $\theta: X \to M$ whose image is essential (i.e., intersects almost every $G$-orbit) and such that $x, y \in X$ are in the same $\Gamma$-orbit if and only if $\theta(x), \theta(y)$ are in the same $G$-orbit. We can then construct a cocycle $\alpha: M \times G \to \Gamma$ such that if $m$, $mg \in \theta(X)$, we have $\theta^{-1}(mg) = \theta^{-1}(m)\alpha(m,g)$. (Cf. [2] and [6, Appendix B].)

LEMMA 2.1. (a) *Suppose $\sigma: \Gamma \to L$ is a homomorphism into a separable group and that $H \subset L$ is a closed subgroup. If $\sigma \circ \alpha: M \times G \to L$ is equivalent to a cocycle taking values in $H$, then there is a measurable $\Gamma$-map $X \to H\backslash L$, where $\Gamma$ acts on $H\backslash L$ via $\sigma$.*

(b) *Suppose $\Gamma_1 \subset \Gamma$ is a subgroup and $\alpha \approx \beta$ where $\beta(M \times G) \subset \Gamma_1$. Then $\Gamma_1$ is of finite index in $\Gamma$.*

PROOF. (a) If $f:M \to L$ satisfies $f(m)\sigma(\alpha(m,g))f(mg)^{-1} = \beta(m,g) \in H$, let $h$ be the projection of $f$ to $H\backslash L$. Then $h(m)\sigma(\alpha(m,g)) = h(mg)$. Then for $x \in X$ and $\lambda \in \Gamma$, we have $\theta(x)g = \theta(x\lambda)$ for some $g$, and $\alpha(\theta(x),g) = \lambda$. If we let $\psi = h \circ \theta$, it follows that $\psi(x)\sigma(\lambda) = \psi(x\lambda)$, i.e., $\psi$ is a $\Gamma$-map.

(b) By (a), there is a measurable $\Gamma$-map $\psi: X \to \Gamma_1\backslash\Gamma$. Since $X$ has a finite $\Gamma$-invariant measure, so will $\Gamma_1\backslash\Gamma$, from which it follows that $\Gamma_1$ is of finite index.

The next lemma is one that is of independent interest and should be of use in other situations. The proof combines the standard proof that a Kazhdan group is finitely generated with some basic known techniques concerning cocycles.

LEMMA 2.2. *Suppose $G$ is a Kazhdan group, $M$ an ergodic $G$-space with finite invariant measure, and $\alpha: M \times G \to \Lambda$ is a cocycle where $\Lambda$ is a countable group. Then $\alpha$ is equivalent to a cocycle taking values in a finitely generated subgroup of $\Lambda$.*

PROOF. Let $\Lambda_n$ be an increasing sequence of finitely generated subgroups of $\Lambda$ whose union is $\Lambda$. Let $\pi_n$ be the unitary representation of $\Lambda$ on $L^2(\Lambda/\Lambda_n)$ given by induction from the identity representation, and let $\pi = \sum^{\oplus}\pi_n$. Then $\pi$ has almost invariant vectors. Let $\beta = \pi \circ \alpha$. Then the proof of [6, Theorem 9.1.1] shows that there is a $\beta$-invariant function $M \to (\sum^{\oplus} L^2(\Lambda/\Lambda_n))_1$ (where the subscript indicates the space of unit vectors), and hence for some fixed $n$ there is a $\beta$-invariant function $M \to (L^2(\Lambda/\Lambda_n))_1$, say $m \to f_m$. For any $\delta > 0$, $S_m = \{y \in \Lambda/\Lambda_n \mid |f_m(y)| \geq \delta\}$ is finite and, for some $\delta$, $S_m$ is non-empty for $m$ in a set of positive measure. By the $\beta$-invariance of $f$ (which we recall asserts that $\beta(g,m)f_m = f_{gm}$), and ergodicity of $G$ acting on $M$, it follows that for some $\delta$ there is a positive integer $k$ such that for almost all $m \in M$ we have $\mathrm{card}(S_m) = k$. Let

$$\Phi: M \to F_k = \{\text{subsets of } \Lambda/\Lambda_n \text{ of cardinality } k\}$$

be the map $m \to S_m$. Then $\Phi$ is an $\alpha$-invariant map for the natural action of $\Lambda$ on $F_k$. Furthermore, $\Lambda$ acts tamely on $F_k$. It follows by the cocycle reduction lemma ([6, Lemma 5.2.11]) that $\alpha$ is equivalent to a cocycle taking all values on a subgroup $\Delta \subset \Lambda$ that is the stabilizer of an element of $F_k$. It follows that $\Delta$ contains a conjugate of $\Lambda_n$ as a subgroup of finite index. Since $\Lambda_n$ is finite generated, it follows that $\Delta$ is as well, proving the lemma.

COROLLARY 2.3. *Under the hypotheses of Theorem* 1.1, $\Gamma$ *is finitely generated.*

PROOF. By Lemma 2.2, $\alpha$ is equivalent to $\beta$ where $\beta(M \times G) \subset \Gamma_1$, where the latter is finitely generated. By Lemma 2.1, $\Gamma_1$ is of finite index in $\Gamma$. Hence, $\Gamma$ is finitely generated as well.

The next lemma is a technical one which will allow us to pass with ease to subgroups of finite index if we wish.

LEMMA 2.4. *Assume the hypotheses of Theorem* 1.1. *If* $\Lambda \subset \Gamma$ *is a subgroup of finite index, then there is an essentially free ergodic action of* $\Lambda$ *with finite invariant measure, say on a space* $Y$, *which is stably orbit equivalent to an essentially free finite measure preserving action of* $G$.

PROOF. First assume $\Lambda$ is normal. Then the space of ergodic components of the $\Lambda$-action on $X$ is finite, and $\Gamma$ transitively permutes these components. Let $Y \subset X$ be one such $\Lambda$-ergodic component, and let $\Delta \subset \Gamma$ be the stabilizer of this component. Then it is easy to see that the orbits of $\Delta$ on $Y$ are in natural bijective correspondence with the orbits of $\Gamma$ on $X$. Hence these actions are stably orbit equivalent, and so the action of $G$ on $M$ is stably orbit equivalent to this action of $\Delta$; i.e., we may assume $\Delta = \Gamma$ or, in other words, that $\Lambda$ acts ergodically on $X$. The action of $\Lambda$ on $X$ is then stably orbit equivalent to the product action of $\Gamma$ on $X \times \Gamma/\Lambda$. But this is, in turn, easily seen to be stably orbit equivalent to the $G$ action on $M \times \Gamma/\Lambda$ given by the cocycle $\alpha$. This is ergodic since $X \times \Gamma/\Lambda$ is, and clearly has a finite invariant measure.

If $\Lambda$ is not assumed to be normal, let $\Lambda_0 \subset \Lambda$ be a subgroup of finite index which is normal in $\Gamma$. By the preceding paragraph, we can find a suitable action of $\Lambda_0$, say on $Y$. The induced action of $\Lambda$, say on $Y \times \Lambda/\Lambda_0$, is readily seen to be stably orbit equivalent to the action of $\Lambda_0$ on $Y$, and hence to a suitable $G$-action.

We now show that the non-finite representation of $\Gamma$ may be taken to be of a special type.

LEMMA 2.5. *Under the hypotheses of Theorem* 1.1, *there is a subgroup* $\Lambda \subset \Gamma$ *of finite index, a finite extension field* $k$ *of* $\mathbf{Q}$, *a connected semisimple algebraic group* $H$ *defined over* $k$, *and a (non-finite) homomorphism* $\lambda : \Lambda \to H(k)$ *with Zariski dense image.*

PROOF. We first claim that there is a non-finite representation $\sigma : \Gamma \to GL(n, \mathbf{Q}^-)$ for some $n$. By inducing a representation from a subgroup of finite index, it suffices to see there is a non-finite representation over $\mathbf{Q}^-$ for some subgroup of finite index in $\Gamma$. However, if every such representation is finite, then [10, Theorem 4.1] implies that every representation of $\Gamma$ over $\mathbf{C}$ is also finite, which contradicts our hypotheses. Let $L$ be the algebraic hull of $\sigma(\Gamma)$. By Lemma 2.4, we may replace $\Gamma$ by a subgroup of finite index if necessary and assume that $L$ is a connected linear algebraic group defined over $\mathbf{Q}^-$. Let $H = L/\mathrm{rad}(L)$, which is then a connected semisimple algebraic group defined over $\mathbf{Q}^{-}$. If the image of $\sigma(\Gamma)$ in $H$ is finite, then $\sigma(\Gamma)$ is a finite extension of a solvable group, and hence is amenable. It follows via Kazhdan's property (namely by [6, Theorem 9.1.1]) that $\sigma \circ \alpha$ is equivalent to a cocycle into a finite subgroup $F$, and hence that $\alpha$ is equivalent to a cocycle into $\sigma^{-1}(F)$. By Lemma 2.1, this is of finite index in $\Gamma$, and hence $F$ is of finite index in $\sigma(\Gamma)$. Thus $\sigma(\Gamma)$ is itself finite, which is impossible. Thus, $\lambda = \mathrm{proj} \circ \sigma$ is a non-finite homomorphism into $H$. Finally, by Lemma 2.2, $\Gamma$ is finitely generated, and hence $\lambda(\Gamma)$ is contained in $H(k)$ for some finite extension $k/\mathbf{Q}$, and since $\lambda(\Gamma)$ is Zariski dense in $H$, $H$ will also be defined over $k$.

We now use superrigidity for $p$-adic valued cocycles to deduce:

LEMMA 2.6. *We can obtain the conclusions of Lemma* 2.5 *with* $k = \mathbf{Q}$, $H$ *with trivial center, and with the image of* $\lambda(\Lambda)$ *discrete in* $H$.

PROOF. Replacing $H$ by the algebraic hull of the image of $\lambda(\Lambda)$ under the embedding $H(k) \to (R_{k/\mathbf{Q}}(H))(\mathbf{Q})$, where the latter is the restriction of scalars [6], we can assume $k = \mathbf{Q}$. Since $\Lambda$ is finitely generated, there will be only a finite set, say $S$, of primes appearing in the denominators of elements of $\lambda(\Lambda)$. We let $H_f$ be the product of the groups $H(\mathbf{Q}_p)$, $p \in S$. The diagonal embedding $H(\mathbf{Q}) \to H \times H_f$ now yields a homomorphism $\Lambda \to H \times H_f$ with discrete image whose projection to the first factor is simply $\lambda$. Let $p : \Lambda \to H_f$ be the projection of this homomorphism to the second factor. By superrigidity for $p$-adic valued cocycles [6, Theorem 5.2.5] the cocycle $p \circ \alpha$ is equivalent to a cocycle taking values in a compact subgroup $C \subset H_f$. By Lemma 2.1, this implies there is a measurable $\Lambda$-map $Y \to H_f/C$. (Here $Y$ is as in the statement of Lemma 2.4 and $\alpha$ is the $\Lambda$-valued cocycle coming from the stable orbit equivalence in 2.4.) This implies there is a finite $\Lambda$-invariant measure on $H_f/C$ and, since $C$ is compact, this in turn implies that the image of $\Lambda$ in $H_f$ is precompact; i.e., the homomorphism $\Lambda \to H \times H_f$ actually has image in $H \times K$ where $K$ is a compact group. It follows that the projection to $H$, i.e. $\lambda$, also has discrete image. Finally, we may clearly divide by the center of $H$ and obtain the lemma.

LEMMA 2.7. *Perhaps by replacing* $\Lambda$ *by a further subgroup of finite index in* $\Lambda$, *there is a homomorphism* $\sigma : \Lambda \to \mathrm{Ad}(G)$ *whose image is a lattice in* $\mathrm{Ad}(G)$.

PROOF. With $H$ as in Lemma 2.6, we can write $H(\mathbf{R}) = H_1 \times H_2$ where $H_1$ is an algebraically connected semisimple real algebraic group with no compact factors and trivial center, and $H_2$ is a compact semisimple Lie group. Let $\lambda_1$ be the composition of $\lambda$ with the projection of $H(\mathbf{R})$ onto the first factor. Then $\lambda_1(\Lambda)$

is a discrete (owing to the compactness of $H_2$) Zariski dense infinite subgroup of $H_1$. In particular, $H_1$ is non-trivial. We claim that the cocycle $\lambda_1 \circ \alpha$ also has algebraic hull $H_1$. (See [6] or [11] for the notion of algebraic hull of a cocycle.) If not, we let $Q \subset H_1$ be the algebraic hull and, by Lemma 2.1, there is a measurable $\Lambda$-map $Y \to H_1/Q$. In particular, there is a $\Lambda$-invariant probability measure on $H_1/Q$. By Chevalley's theorem [6], $H_1/Q$ can be embedded as an orbit in projective space for a rational representation of $H_1$, and hence by [6, Theorem 3.2.4] the stabilizers of probability measures on $H_1/Q$ are real algebraic subgroups of $H_1$. Since $\lambda_1(\Lambda)$ is Zariski dense in $H_1$, there is an $H_1$-invariant probability measure on $H_1/Q$. By the Borel density theorem [6] we then deduce that $Q = H_1$.

Applying superrigidity for real cocycles [6, Theorem 5.2.5], we deduce that there is a rational local isomorphism $\pi : G \to H_1$ such that $\lambda_1 \circ \alpha$ is equivalent to the cocycle $\alpha_\pi(m,g) = \pi(g)$. Thus, there is a measurable function $f : M \to H_1$ such that $f(m)^{-1}\lambda_1(\alpha(m,g))f(mg) = \pi(g)$. Let $q$ be the projection $H_1 \to \lambda_1(\Lambda)\backslash H_1$. (We recall that $\lambda_1(\Lambda)$ is discrete in $H_1$.) It follows that $h = q \circ f$ satisfies $h(mg) = h(m)\pi(g)$, i.e., $h : M \to \lambda_1(\Lambda)\backslash H_1$ is a $G$-map. Hence, there is a $G$-invariant measure on $\lambda_1(\Lambda)\backslash H_1$ with the action defined via $\pi$. Since $\pi$ is a local isomorphism, its image is a subgroup of finite index, and it follows that $\lambda_1(\Lambda)$ is a lattice in $H_1$. Passing to a subgroup of finite index, we can assume the image is a lattice in $\mathrm{Ad}(G)$.

To complete the proof of Theorem 1.1, it suffices to prove:

LEMMA 2.8. *Let $\sigma$ and $\Lambda$ be as in Lemma 2.7, and let $\alpha : M \times G \to \Lambda$ be a cocycle coming from a stable orbit equivalence as above. Then* $\ker(\sigma)$ *is finite.*

PROOF. Let $\Delta = \sigma(\Lambda)$, and consider the product action of $\Lambda$ on $Y \times \Delta$. This action is stably orbit equivalent to the action of $\ker(\sigma)$ on $Y$. Since the action of $\Lambda$ on $Y$ is essentially free and preserves a finite measure, to see that $\ker(\sigma)$ is finite it suffices to see that the equivalence relation defined by the action of $\ker(\sigma)$ on $Y$ is type I (in the sense of [2] or [3]), and hence that the action of $\Lambda$ on $Y \times \Delta$ is type I. It is straightforward to check that this action in turn is stably orbit equivalent to the action of $G$ on $M \times \Delta$, where the action is given by the cocycle $\sigma \circ \alpha : M \times G \to \Delta$. It therefore suffices to see that this action is type I. Let $i : \Delta \to \mathrm{Ad}(G)$ be the inclusion. A straightforward application of Fubini's theorem shows that it suffices to see that the action of $G$ on $M \times \mathrm{Ad}(G)$ given by $i \circ \sigma \circ \alpha$ is of type I. However, as we have seen, $i \circ \sigma \circ \alpha$ is equivalent to the cocycle $\alpha_\pi$ where $\pi : G \to \mathrm{Ad}(G)$ is a smooth homomorphism. Thus, it suffices to see that the product action of $G$ on $M \times \mathrm{Ad}(G)$ is type I, where $g \in G$ acts on $\mathrm{Ad}(G)$ via translation by

$\pi(g)$. This action is stably orbit equivalent to the action of $\ker(\pi)$ on $M$, which is type I since $\ker(\pi)$ is finite.

This completes the proof of Theorem 1.1.

## 3. Proof of the corollaries

Corollary 1.2 follows from the theorem by applying the latter to the action of $G$ induced from the given action of $\Lambda$. Corollary 1.3 follows from the definition of residual finiteness, namely, given a finite subgroup, one can find a finite index subgroup whose intersection with the finite group is trivial. Corollary 1.5 follows from basic facts about transverse structures for foliations, and the relation of holonomy to orbit equivalence as explained in [8] and the references therein.

We now consider Corollary 1.4. Consider the situation in (a), where we allow the more general condition that $H$ is connected semisimple with finite center and $\Lambda \subset H$ is an irreducible lattice. Suppose $\Gamma$ acts on $T$ essentially freely, giving the equivalence relation on $T$. By Theorem 1.1, there is a finite normal subgroup $N \subset \Gamma$ such that $\Gamma/N$ is isomorphic to a lattice $\Delta \subset \mathrm{Ad}(G) \approx G$. Then $N \subset \Gamma$, being a finite group, acts tamely on $T$, and there is an action of $\Delta$ on $T/N$ that is stably orbit equivalent to the action of $\Gamma$ on $T$. It follows that the action of $G$ induced by this action of $\Delta$ is stably orbit equivalent to the original action of $G$ on $H/\Lambda$, and hence these actions are orbit equivalent. By [6, Theorem 5.2.1] these actions are in fact conjugate modulo an automorphism of $G$. It follows that modifying the action of $G$ on $H/\Lambda$ by an automorphism of $G$, there is a measurable $G$-map $H/\Lambda \to G/\Delta$ (since such a map exists for an induced action by definition). But by results of Witte [5, Corollary 5.6] this implies that as a $G$-space $G/\Delta$ is measurably isomorphic to $K\backslash H/\Lambda'$ where $K$ is compact and $\Lambda' \supset \Lambda$ is another lattice. This is clearly impossible. Taking $H$ to be simple yields (a), and taking $H = G \times G$ yields (b) by the observation that the equivalence relation on the set $A$ in the statement of 1.4(b) is stably orbit equivalent to the equivalence relation defined by $\Lambda$ acting on $G$, which in turn is stably orbit equivalent to the action of $G$ on $(G \times G)/\Lambda$ given by the embedding in the other component (i.e., other than the component onto which we are projecting $\Lambda$).

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